MIMO-OFDM Crystallized Rate Regions

Pawel Sroka[1], Adrian Kliks[1], and Merouane Debbah[2]
[1]Poznan University of Technology, Chair of Wireless Communications, Poznań, Poland
e-mail: {akliks, psroka}@et.put.poznan.pl
[2] SUPELEC, Alcatel-Lucent Chair on Flexible Radio, GIF SUR YVETTE, France
e-mail: merouane.debbah@supelec.fr

Abstract

In this paper, we introduce the generalization of the concept of crystallized rate regions for MIMO-OFDM transmission. The extension from the OFDM and MIMO case to MIMO-OFDM scenario of the time-sharing convex hull of achievable rates is discussed and a new definition of the cost function for the rate region game is derived. Based on the new game definition, the simulation results for the MIMO-OFDM case are presented.

1 Introduction

The future wireless systems are characterized by decreasing range of the transmitters as higher transmit frequencies are to be utilized, and the impact of interference on the overall system performance is very significant. Hence, interference mitigation between transmiter-receiver pairs is of great importance in order to improve the achievable data rates.

The Multiple Input Multiple Output (MIMO) technology has enabled further increase in system throughput. Moreover, the utilization of spatial diversity thanks to MIMO technology opens new possibilities of interference mitigation [1, 2]. On the other hand, Orthogonal Frequency Division Multiplexing (OFDM) technique has been recently widely considered as a key and sophisticated solution in wireless and wired communication systems in particular in the presence of deep fading [3]. Clearly, the synergy of the abovementioned techniques, called MIMO-OFDM transmission, offers new possibilities in increasing the achievable channel capacity and system robustness in the presence of high delay and Doppler spread.

This paper extends the idea of crystallized rate regions presented in [4] and adapted to the MIMO scenario in [5] to the MIMO-OFDM interference channel, which will be outlined in section 2. The application of the game-theoretic correlated equilibrium concept [6] to the rate region problem is considered. Moreover, a Vickey-Clarke-Groves (VCG) auction utility formulation and the regret-matching learning algorithm [4, 5] are employed to demonstrate the application of the considered concept for the 2-user MIMO-OFDM channel, what is referenced in Section 3. Finally, Section 4 presents the considered ideas through simulation, and Section 5 draws the conclusions.

2 Crystallized Rate Regions for MIMO-OFDM Transmission

In this section, we present the generalization of the concept of crystallized rate region in the context of the OFDM-MIMO transmissions. We start by defining the channel model under study. Then, we analyse the achievable rate regions for the interference MIMO-OFDM channel, when interference is treated as Gaussian noise. Finally, the generalized definition of the rate regions for the MIMO-OFDM transmission are presented.

2.1 System model for 2-user Interference MIMO-OFDM channel

The 2-user 2-cell uplink interference MIMO-OFDM channel is considered in this paper. Each Mobile Terminal (MT) is equipped with N_t transmit antennas and each Base Station (BS) has N_r receive antennas. Moreover, user i can transmit data with the maximum total power equal to $P_{i,max}$. The OFDM transmission with M subcarriers is considered. All MTs are assumed to have perfect channel knowledge. In order to ease the analysis, we limit the number of frequency bins to 2 in our derivation. In the general 2 user case, user i - which suffers from the interference from user j - transmits the signal vector $X_i \in \mathbb{C}^{\,N_t M}$ through the multipath interference channel, where channel transfer matrix is defined as $\mathbf{H} \in \mathbb{C}^{\,2N_r M \times 2N_t M}$ where

$$\mathbf{H} = \begin{pmatrix} \mathbf{H}_{11} & \mathbf{H}_{12} \\ \mathbf{H}_{21} & \mathbf{H}_{22} \end{pmatrix}, \qquad \mathbf{H}_{i,j} \in \mathbb{C}^{\,N_r M \times N_t M} \tag{1}$$

and $\mathbf{H}_{i,j} = diag\left\{\mathbf{H}_{i,j}^{(m)}\right\}$, $1 \leq m \leq M$, is the block diagonal channel transfer function between transmitter i and receiver j, and $diag\{\cdot\}$ denotes the diagonal matrix. The channel transfer matrix $\mathbf{H}_{i,j}^{(m)}$ corresponds to the mth subcarrier and is defined as $\mathbf{H}_{i,j}^{(m)} = \left\{h_{k,l}^{(m,i,j)} \in \mathcal{CN}(0,1), 1 \leq l \leq N_r, 1 \leq k \leq N_t\right\}$, where k and l denote the transmit antenna at the i-th MT and the receive antenna at the j-th BS, respectively. Additive White Gaussian Noise (AWGN) of zero mean and variance σ^2 is added to the received signal. The received signal at BS i can be defined as $y_i = H_i i \cdot x_i + H_j i \cdot x_j + z_i$, where x_i is the signal transmited by MT i and z_i is the additive noise. With the channel known at the transmitter, the throughput can be improved by applying some well-known MIMO transmission techniques, such as the linearization (diagonalization) of the channel by the means of Eigenvalue Decomposition (EVD) or Singular Value Decomposition (SVD) [7] (denoted hereafter as SVD-MIMO), or transmit and receive beamforming, with the transmit precoding based on the predefined codebook. The following codebook generation methods have been considered: LTE codebook [8], Per-User Unitary Rate Control (PU^{2}RC) MIMO precoding [9], and a random-beamforming approach [10], where the set of N precoders is obtained in a random manner (denoted hereafter as RAN-N).

When the set of randomly generated beamformers is used, the set of receive beamformers has to be calculated at the receiver. Different criteria can be used, with the following ones considered in our simulations: Zero-Forcing (ZF), Minimum Mean Squared Error (MMSE) and Maximum-Likelihood (ML) [7]. In the following description, the abbreviations based on the combination of certain transmit and receive beamforming techniques is used, e.g. for ZF-MIMO-LTE Long Term Evolution (LTE) codebook is used for transmit precoding and the receiver is designed using the ZF criterion.

When treating interference as noise, the achievable rates for 2-user interference MIMO channel are defined as in [11]. Taking into account the transmit and receive MIMO processing the rate of user i can be defined as:

$$R_i\left(\mathbf{Q}_i, \mathbf{Q}_j\right) = log_2(det(\mathbf{I} + \mathbf{u}_i^* \mathbf{H}_{ii} \mathbf{v}_i \mathbf{Q}_i \mathbf{v}_i^* \mathbf{H}_{ii}^* \mathbf{u}_i \cdot(\sigma^2 \mathbf{u}_i^* \mathbf{u}_i + \mathbf{u}_i^* \mathbf{H}_{ji} \mathbf{v}_j \mathbf{Q}_j \mathbf{v}_j \mathbf{H}_{ji}^* \mathbf{u}_i)^{-1})), \tag{2}$$

where R_i is the rate of user i, j stands for the interfering user, $(\cdot)^*$ denotes matrix transpose conjugate, $\mathbf{u}_i$ and $\mathbf{v}_i$ denote the ith user's set of receive and transmit beamformers, respectively, and $\mathbf{Q}_i$ is the ith user data covariance matrix, i.e. $E\left\{X_i X_i^*\right\} = \mathbf{Q_i}$ and $tr(\mathbf{Q}_i) \leq P_{i,max}$. Hence, we define the rate region as $\Re = \bigcup \left\{(R_1(\mathbf{Q}_1, \mathbf{Q}_2), R_2(\mathbf{Q}_1, \mathbf{Q}_2))\right\}$.

For the sake of simplicity we will limit our further analysis to the 2×2 MIMO case.

2.2 Achievable Rate Regions in a case of interference MIMO-OFDM channel

The rate region for the general n-user SISO channel has been found to be the convex hull of the union of n hypersurfaces [12], which means that the rate regions entirely encloses a straight line that connects any two points which lie within the rate region bounds. In the 2-user case the rate regions can be easily represented as the surface limited by the horizontal and vertical axes and the boundaries of the 2-dimensional hypersurface (straight lines). Let us stress that the same conclusions can be drawn for the MIMO case. Thus, we will now discuss the achievable rate regions for the interference MIMO-OFDM channel. It is now assumed that the transmit and receive beamformers are computed in an (sub)-optimal way by the means of SVD of the channel transfer function. The channel model considered in the investigation was an extension to MIMO-OFDM case of one given in [5]. The rate region obtained for the interference SVD-MIMO-OFDM channel, is shown in Fig. 1.

One can define five characteristic points on the border of the rate region, i.e. points A, B, C, D and E. Specifically, point A describes the situation, where the first user transmits with the maximum power and Q_1 is chosen such that $Q_1 = \arg\max_{\tilde{\mathbf{Q}}_1} R_1(\tilde{\mathbf{Q}}_1, \mathbf{Q}_2 = \mathbf{0})$. Point B corresponds to the situation, where first user transmits with the maximum power whereas the transmit power of the second user is lower than maximal value, and the distribution of the power among the antennas and subcarriers is optimal in the sum-rate sense, i.e.

$$(Q_1, Q_2) = \arg\max_{\tilde{\mathbf{Q}}_1, \tilde{\mathbf{Q}}_2} \left(R_1(\tilde{\mathbf{Q}}_1, \tilde{\mathbf{Q}}_2) + R_2(\tilde{\mathbf{Q}}_1, \tilde{\mathbf{Q}}_2)\right)$$
$$subject \quad to \quad tr(\mathbf{Q}_1) = P_{1,max}, tr(\mathbf{Q}_2) = p \leq P_{2,max},$$

where $P_{i,max}$ is the maximum allowed transmit power of user i.

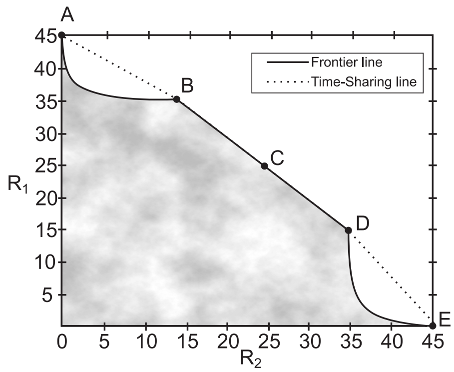

Figure 1: Rate region obtained for the fixed MIMO-OFDM scenario

Points D and E can be defined by symmetry to A and B. Finally, C corresponds to the situation where both users transmit with maximum power. One can observe, that the achievable rate region for the two user 2×2 MIMO-OFDM case is not convex, thus the time-sharing (see subsection 2.3) approach seems to be right way for system capacity improvement. The potential time-sharing lines are also presented in Fig. 1.

2.3 Crystallized Rate Regions and Time-Sharing Coefficients for the MIMO-OFDM transmission

The idea of the Crystallized Rate Regions has been introduced in [4] and can be understood as approximating of the achievable rate regions by the convex time-sharing hull, where the potential curves between characteristic points (e.g. A to E in Fig. 1) are replaced by the straight lines connecting these points.

In the 2-user 2×2 MIMO-OFDM channel with 2 subcarriers there exist 256 characteristic points, which refer to any particular combination of the possible strategies. In general, for the n-user N_txN_r MIMO-OFDM case with M subcarriers, there exist $(N_t + 2)^{n \cdot M}$ points, i.e. the i-th user can put all power to one antenna ($(N_t)^M$ possibilities), divide the power equally among the antennas (one possibility) or be silent (one possibility).

Following the approach proposed in [4] and extended in [5], one can state that the power control problem can be substituted with finding the appropriate time-sharing coefficients of the $(N_t + 2)^{n \cdot M}$ corner points. The time-sharing coefficients can be obtained using a game-theoretic approach considering the application of the correlated equilibrium concept [6], as proposed in [4, 5].

3 Mechanism Design and Regret-Matching Learning Algorithm

The rate optimization over the interference channel requires two major issues to be coped with: first, ensure the system convergence to the desired point, that can be achieved using an auction utility function; second, a distributive solution is necessary to achieve the equilibrium, such as the proposed regret-matching algorithm. In order to fulfill these requirements the Vickrey-Clarke-Groves (VCG) auction mechanism design is employed in the Regret-Matching algorithm first derived in [12] and modified for the MIMO case in [5]. Hereafter we apply similar modifications to the ones proposed in [5] in order to learn in a distributive fashion how to achieve the correlated equilibrium set in solving the VCG auction.

4 Simulation Results

To validate the applicability of the proposed idea to the power control problem, system-level snapshot simulations of the 2-user 2×2 MIMO-OFDM system with 2 subcarriers have been performed. Specific rate values obtained for the considered MIMO implementations are given in Table 1. The results have obtained for all RAN-8 scenarios (i.e. ZF, MMSE and ML and when the codebook size is equal to 8) at a confidence level of 99%. One can notice that for all cases, when the Regret Matching algorithm has been applied, the obtained rates are similar to each other and relatively close to the optimal solution, which corresponds to point C in Fig. 1. Only for the ZF/MMSE-MIMO cases when the random beamforming approach has been used the averaged results are significantly worse because of high dependency of algorithms efficiency on the actual set of transmit beamform-

Table 1: Average achieved rates

MIMO scheme	User 1	User2
SVD	24.11	26.27
ZF-RAN-8	13.73 (max. 23.12)	13.64 (max. 22.17)
MMSE-RAN-8	13.7 (max. 23.12)	13.61 (max. 22.72)
ML-RAN-8	23.94	21.06
ZF-LTE	23.94	23.97
MMSE-LTE	23.91	23.97
ML-LTE	23.98	23.99
ZF-PU^{2}RC-8	23.90	23.97
MMSE-PU^{2}RC-8	23.90	23.97
ML-PU^{2}RC-8	23.94	23.99

ers, which are generated on a random manner. If the set of beamformers is well-defined (i.e. at least one precoder matches closely the actual channel conditions for i-th user) the achieved rate is also close to the optimal point (see the maximum obtained values for one particular channel realization).

The efficiency of the random beamforming technique depends on the number of precoders. However, the higher number of precoders the higher the complexity of the algorithm. Thus, in order to present the relation between the random beamforming technique efficiency and the codebook size the computer simulations of ZF-RAN-MIMO and MMSE-RAN-MIMO configurations have been carried out for the considered particular channel realization. The results, presented in Fig. 2, have been obtained for various codebook realizations at confidence level of 99% for each codebook size. One can observe that the obtained rate for both users increase approximately logarithmically as the number of precoders increase.

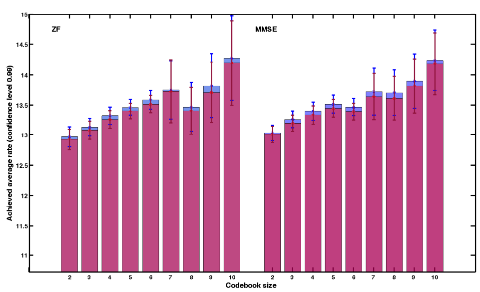

Figure 2: Achieved average rate versus codebook size (user 1 - blue, user 2 - red)

5 Conclusions

In this paper the idea of crystallized rate regions, introduced first in the context of finding the capacity of the SISO interference channel in [4], and further developed for the MIMO case in [5], has been extended to the MIMO-OFDM interference channels. The applicability of the correlated equilibrium instead of the well-known Nash equilibrium has been verified to be adequate for the case of 2-user MIMO-OFDM transmission. A VCG auction utility function and the regret-matching algorithm derived in [5] have been applied to the generalized MIMO-OFDM case. Simulation results for the selected 2-user scenarios proved the correctness of application of the crystallized rate regions concept to the general MIMO-OFDM scenarios.

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